

Johann Georg Neidhardt, 1732

Johann Georg Neidhardt's 21 temperaments of 1732

Neidhardt, Johann Georg. *Gänzlich erschöpfte, mathematische Abtheilungen des diatonisch-chromatischen, temperirten Canonis Monochordi*, Königsberg and Leipzig 1732. Chapters VI-VII, presenting 21 temperament examples.

Facsimiles of these two chapters are available at <http://harpsichords.pbwiki.com/Tuning>

The following charts present a series of consistently-calculated measurements of Neidhardt's 21 temperament examples. These are calculated in each case by taking his columns of 5th (*Quintae*) as fragments of the Pythagorean comma.

Neidhardt's typographic conventions of ^ and V indicate narrow and wide 5th, respectively.

In the following charts, measurements are provided as follows:

- Semitones and tones are measured in cents.
- Minor 3^{rds} are measured as a percentage of the syntonic comma from purity (6:5 ratio).
- Major 3^{rds} are measured as a percentage of the syntonic comma from purity (5:4 ratio).
- Within each stack of three major 3^{rds}, the distribution of the *diesis* is given both as a percentage and as a set of three integers (which must add up to 21: the size of three syntonic commas less one Pythagorean comma, within a pure 2:1 octave). These integers are on a scale where 1 = 1/12 Pythagorean comma.
- 5th (and by implication also their inversions, 4^{ths}) are measured as comma-fragment discrepancies from purity (3:2): both as Pythagorean comma and syntonic comma, in different rows of the chart. Another row gives this same information on a 720-unit scale, the "Temperament Units" developed by John Bromaugh: where a full Pythagorean comma is 720 units, and a full syntonic comma is 660 units.
- The positions of each individual note are also given as cent offsets from the regularity of 1/6 Pythagorean comma temperament, 1/6 syntonic comma temperament, and equal temperament (1/12 Pythagorean). The equal-temperament offsets are given in two separate rows, keeping either C or A at 0.0; this is for use by anyone who might wish to try these with an electronic tuning device that generates cent offsets.
- The melodic characters of major and minor tetrachords are given as cent measurements of their semitones and tones. The major tetrachord is Ut-Re-Mi-Fa, and the minor tetrachord is Re-Mi-Fa-Sol.

Johann Georg Neidhardt, 1732

A technical note: as Neidhardt does not distinguish here between the Pythagorean and syntonic commas, all of his own calculated measurements of major 3rds (in his own published tables) are off by 1 unit. In his calculations for the sizes of 3rds he should have subtracted from a total of 11, instead of from 12 as explained at the beginning of chapter VI. Four pure 5^{ths} in succession generate a major 3rd that is 11/12 of a Pythagorean comma sharp (i.e. the approximate size of the *syntonic* comma); not 12/12. Georg Andreas Sorge's later work corrected this discrepancy of measurement, and the correctly calculated values are given here at the lower left section of each chart, as fragments of the Pythagorean comma.

- Bradley Lehman, October 2006

Johann Georg Neidhardt, 1732

Neidhardt's chapter VI begins with this trivial example, placing all of the Pythagorean comma into one 5th:

Neidhardt (1732) Pythagorean example												
	E_b	B_b	F	C	G	D	A	E	B	F[#]	C[#]	G[#]
Fifth in TU	0	0	0	-720	0	0	0	0	0	0	0	0
Fifth in PC				- 1/1								
Fifth in SC				- 12/11								
Maj3rd % SC	-9.1%	-9.1%	-9.1%	-9.1%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
Diesis portion	-4.8%	-4.8%	-4.8%	-4.8%	52.4%	52.4%	52.4%	52.4%	52.4%	52.4%	52.4%	52.4%
Min3rd % SC	-100.0%	-100.0%	-100.0%	-100.0%	9.1%	9.1%	9.1%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%
Tone c	203.9	203.9	180.4	180.4	203.9	203.9	203.9	203.9	203.9	203.9	203.9	203.9
Semitone c	90.2	90.2	90.2	90.2	113.7	113.7	113.7	113.7	113.7	90.2	90.2	90.2
Enharmonic	<i>D[#]</i>	<i>A[#]</i>	<i>E[#]</i>	<i>B[#]</i>	<i>F_x</i>	<i>C_x</i>	<i>B_{bb}</i>	<i>F_b</i>	<i>C_b</i>	<i>G_b</i>	<i>D_b</i>	<i>A_b</i>
Enharm err % PC	50.0%	66.7%	83.3%	100.0%	16.7%	33.3%	-150.0%	-133.3%	-116.7%	-100.0%	-83.3%	-66.7%
d 1/6PC (C) c	-11.7	-7.8	-3.9	0.0	-19.6	-15.6	-11.7	-7.8	-3.9	0.0	3.9	7.8
Error % PC	-50.0%	-33.3%	-16.7%	0.0%	-83.3%	-66.7%	-50.0%	-33.3%	-16.7%	0.0%	16.7%	33.3%
	E_b	B_b	F	C	G	D	A	E	B	F[#]	C[#]	G[#]
d 1/6SC (C) c	-10.8	-7.2	-3.6	0.0	-19.9	-16.3	-12.7	-9.1	-5.5	-2.0	1.6	5.2
d ET (C) c	-5.9	-3.9	-2.0	0.0	-21.5	-19.6	-17.6	-15.6	-13.7	-11.7	-9.8	-7.8
d ET (A) c	11.7	13.7	15.6	17.6	-3.9	-2.0	0.0	2.0	3.9	5.9	7.8	9.8

Major tetrachord	E _b -F-G-A _b	B _b -C-D-E _b	F-G-A-B _b	C-D-E-F	G-A-B-C	D-E-F [#] -G	A-B-C [#] -D	E-F [#] -G [#] -A	B-C [#] -D [#] -E	F [#] -G [#] -A [#] -B	D _b -E _b -F-G _b	A _b -B _b -C-D _b
Ut-Re	203.9	203.9	180.4	180.4	203.9	203.9	203.9	203.9	203.9	203.9	203.9	203.9
Re-Mi	180.4	180.4	203.9	203.9	203.9	203.9	203.9	203.9	203.9	203.9	203.9	203.9
Mi-Fa	113.7	113.7	113.7	113.7	113.7	90.2	90.2	90.2	90.2	90.2	90.2	90.2
Minor tetrachord	E _b -F-G _b -A _b	B _b -C-D _b -E _b	F-G-A _b -B _b	C-D-E _b -F	G-A-B _b -C	D-E-F-G	A-B-C-D	E-F [#] -G-A	B-C [#] -D-E	F [#] -G [#] -A-B	C [#] -D [#] -E-F [#]	G [#] -A [#] -B-C [#]
Re-Mi	203.9	203.9	180.4	180.4	203.9	203.9	203.9	203.9	203.9	203.9	203.9	203.9
Mi-Fa	90.2	90.2	113.7	113.7	113.7	113.7	113.7	90.2	90.2	90.2	90.2	90.2
Fa-Sol	203.9	203.9	203.9	203.9	203.9	180.4	180.4	203.9	203.9	203.9	203.9	203.9

Handling diesis	Ab-C 11	Db-F 11	F [#] -A [#] 11	B-D [#] 11
within the octave	E-G [#] 11	A-C [#] 11	D-F [#] 11	G-B 11
(portions of 21)	C-E -1	F-A -1	B _b -D -1	E _b -G -1

Johann Georg Neidhardt, 1732

He next presents a chart of 12 temperament examples, “Quinten-Circuls”, distributing the Pythagorean comma in various manners:

Neidhardt (1732) 5th-circle #1 (equal temperament)												
	E_b	B_b	F	C	G	D	A	E	B	F_#	C_#	G_#
Fifth in TU	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60
Fifth in PC	- 1/12	- 1/12	- 1/12	- 1/12	- 1/12	- 1/12	- 1/12	- 1/12	- 1/12	- 1/12	- 1/12	- 1/12
Fifth in SC	- 1/11	- 1/11	- 1/11	- 1/11	- 1/11	- 1/11	- 1/11	- 1/11	- 1/11	- 1/11	- 1/11	- 1/11
Maj3rd % SC	63.6%	63.6%	63.6%	63.6%	63.6%	63.6%	63.6%	63.6%	63.6%	63.6%	63.6%	63.6%
Diesis portion	33.3%	33.3%	33.3%	33.3%	33.3%	33.3%	33.3%	33.3%	33.3%	33.3%	33.3%	33.3%
Min3rd % SC	-72.7%	-72.7%	-72.7%	-72.7%	-72.7%	-72.7%	-72.7%	-72.7%	-72.7%	-72.7%	-72.7%	-72.7%
Tone c	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0
Semitone c	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Enharmonic	<i>D_#</i>	<i>A_#</i>	<i>E_#</i>	<i>B_#</i>	<i>F_x</i>	<i>C_x</i>	<i>B_{bb}</i>	<i>F_b</i>	<i>C_b</i>	<i>G_b</i>	<i>D_b</i>	<i>A_b</i>
Enharm err % PC	75.0%	83.3%	91.7%	100.0%	108.3%	116.7%	-75.0%	-66.7%	-58.3%	-50.0%	-41.7%	-33.3%
d 1/6PC (C) c	-5.9	-3.9	-2.0	0.0	2.0	3.9	5.9	7.8	9.8	11.7	13.7	15.6
Error % PC	-25.0%	-16.7%	-8.3%	0.0%	8.3%	16.7%	25.0%	33.3%	41.7%	50.0%	58.3%	66.7%
	E_b	B_b	F	C	G	D	A	E	B	F_#	C_#	G_#
d 1/6SC (C) c	-4.9	-3.3	-1.6	0.0	1.6	3.3	4.9	6.5	8.1	9.8	11.4	13.0
d ET (C) c	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
d ET (A) c	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Major tetrachord	E _b -F-G-A _b	B _b -C-D-E _b	F-G-A-B _b	C-D-E-F	G-A-B-C	D-E-F _# -G	A-B-C _# -D	E-F _# -G _# -A	B-C _# -D _# -E	F _# -G _# -A _# -B	D _b -E _b -F-G _b	A _b -B _b -C-D _b
Ut-Re	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0
Re-Mi	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0
Mi-Fa	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Minor tetrachord	E _b -F-G _b -A _b	B _b -C-D _b -E _b	F-G-A _b -B _b	C-D-E _b -F	G-A-B _b -C	D-E-F-G	A-B-C-D	E-F _# -G-A	B-C _# -D-E	F _# -G _# -A-B	C _# -D _# -E-F _#	G _# -A _# -B-C _#
Re-Mi	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0
Mi-Fa	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Fa-Sol	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0

Handling diesis	A _b -C 7	D _b -F 7	F _# -A _# 7	B-D _# 7
within the octave	E-G _# 7	A-C _# 7	D-F _# 7	G-B 7
(portions of 21)	C-E 7	F-A 7	B _b -D 7	E _b -G 7

	E♭	B♭	F	C	G	D	A	E	B	F♯	C♯	G♯
Fifth in TU	120	-120	-120	-120	120	-120	-120	-120	120	-120	-120	-120
Fifth in PC	1/6	- 1/6	- 1/6	- 1/6	1/6	- 1/6	- 1/6	- 1/6	1/6	- 1/6	- 1/6	- 1/6
Fifth in SC	2/11	- 2/11	- 2/11	- 2/11	2/11	- 2/11	- 2/11	- 2/11	2/11	- 2/11	- 2/11	- 2/11
Maj3rd % SC	63.6%	63.6%	63.6%	63.6%	63.6%	63.6%	63.6%	63.6%	63.6%	63.6%	63.6%	63.6%
Diesis portion	33.3%	33.3%	33.3%	33.3%	33.3%	33.3%	33.3%	33.3%	33.3%	33.3%	33.3%	33.3%
Min3rd % SC	-45.5%	-81.8%	-81.8%	-81.8%	-45.5%	-81.8%	-81.8%	-81.8%	-45.5%	-81.8%	-81.8%	-81.8%
Tone c	203.9	196.1	196.1	203.9	203.9	196.1	196.1	203.9	203.9	196.1	196.1	203.9
Semitone c	102.0	94.1	102.0	102.0	102.0	94.1	102.0	102.0	102.0	94.1	102.0	102.0
Enharmonic	<i>D♯</i>	<i>A♯</i>	<i>E♯</i>	<i>B♯</i>	<i>F×</i>	<i>C×</i>	<i>Bbb</i>	<i>Fb</i>	<i>Cb</i>	<i>Gb</i>	<i>Db</i>	<i>Ab</i>
Enharm err % PC	66.7%	100.0%	100.0%	100.0%	100.0%	133.3%	-66.7%	-66.7%	-66.7%	-33.3%	-33.3%	-33.3%
d 1/6PC (C) c	-7.8	0.0	0.0	0.0	0.0	7.8	7.8	7.8	7.8	15.6	15.6	15.6
Error % PC	-33.3%	0.0%	0.0%	0.0%	0.0%	33.3%	33.3%	33.3%	33.3%	66.7%	66.7%	66.7%
Primary function	E♭	B♭	F	C	G	D	A	E	B	F♯	C♯	G♯
d 1/6SC (C) c	-6.8	0.7	0.3	0.0	-0.3	7.2	6.8	6.5	6.2	13.7	13.4	13.0
d ET (C) c	-2.0	3.9	2.0	0.0	-2.0	3.9	2.0	0.0	-2.0	3.9	2.0	0.0
d ET (A) c	-3.9	2.0	0.0	-2.0	-3.9	2.0	0.0	-2.0	-3.9	2.0	0.0	-2.0

Major tetrachord	E♭-F-G-A♭	B♭-C-D-E♭	F-G-A-B♭	C-D-E-F	G-A-B-C	D-E-F♯-G	A-B-C♯-D	E-F♯-G♯-A	B-C♯-D♯-E	F♯-G♯-A♯-B	D♭-E♭-F-G♭	A♭-B♭-C-D♭
Ut-Re	203.9	196.1	196.1	203.9	203.9	196.1	196.1	203.9	203.9	196.1	196.1	203.9
Re-Mi	196.1	203.9	203.9	196.1	196.1	203.9	203.9	196.1	196.1	203.9	203.9	196.1
Mi-Fa	102.0	94.1	102.0	102.0	102.0	94.1	102.0	102.0	102.0	94.1	102.0	102.0
Minor tetrachord	E♭-F-G♭-A♭	B♭-C-D♭-E♭	F-G-A♭-B♭	C-D-E♭-F	G-A-B♭-C	D-E-F-G	A-B-C-D	E-F♯-G-A	B-C♯-D-E	F♯-G♯-A-B	C♯-D♯-E-F♯	G♯-A♯-B-C♯
Re-Mi	203.9	196.1	196.1	203.9	203.9	196.1	196.1	203.9	203.9	196.1	196.1	203.9
Mi-Fa	102.0	102.0	102.0	94.1	102.0	102.0	102.0	94.1	102.0	102.0	102.0	94.1
Fa-Sol	196.1	196.1	203.9	203.9	196.1	196.1	203.9	203.9	196.1	196.1	203.9	203.9

Handling dies	Ab-C 7	Db-F 7	F#-A# 7	B-D# 7
within the octave	E-G# 7	A-C# 7	D-F# 7	G-B 7
(portions of 21)	C-E 7	F-A 7	Bb-D 7	Eb-G 7

Johann Georg Neidhardt, 1732

Neidhardt (1732) 5th-circle #3												
	E_b	B_b	F	C	G	D	A	E	B	F_#	C_#	G_#
Fifth in TU	-120	0	-120	0	-120	0	-120	0	-120	0	-120	0
Fifth in PC	- 1/6		- 1/6		- 1/6		- 1/6		- 1/6		- 1/6	
Fifth in SC	- 2/11		- 2/11		- 2/11		- 2/11		- 2/11		- 2/11	
Maj3rd % SC	63.6%	63.6%	63.6%	63.6%	63.6%	63.6%	63.6%	63.6%	63.6%	63.6%	63.6%	63.6%
Diesis portion	33.3%	33.3%	33.3%	33.3%	33.3%	33.3%	33.3%	33.3%	33.3%	33.3%	33.3%	33.3%
Min3rd % SC	-81.8%	-63.6%	-81.8%	-63.6%	-81.8%	-63.6%	-81.8%	-63.6%	-81.8%	-63.6%	-81.8%	-63.6%
Tone c	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0
Semitone c	98.0	102.0	98.0	102.0	98.0	102.0	98.0	102.0	98.0	102.0	98.0	102.0
Enharmonic	<i>D_#</i>	<i>A_#</i>	<i>E_#</i>	<i>B_#</i>	<i>F_x</i>	<i>C_x</i>	<i>B_{bb}</i>	<i>F_b</i>	<i>C_b</i>	<i>G_b</i>	<i>D_b</i>	<i>A_b</i>
Enharm err % PC	83.3%	83.3%	100.0%	100.0%	116.7%	116.7%	-66.7%	-66.7%	-50.0%	-50.0%	-33.3%	-33.3%
d 1/6PC (C) c	-3.9	-3.9	0.0	0.0	3.9	3.9	7.8	7.8	11.7	11.7	15.6	15.6
Error % PC	-16.7%	-16.7%	0.0%	0.0%	16.7%	16.7%	33.3%	33.3%	50.0%	50.0%	66.7%	66.7%
Primary function	E_b	B_b	F	C	G	D	A	E	B	F_#	C_#	G_#
d 1/6SC (C) c	-2.9	-3.3	0.3	0.0	3.6	3.3	6.8	6.5	10.1	9.8	13.4	13.0
d ET (C) c	2.0	0.0	2.0	0.0	2.0	0.0	2.0	0.0	2.0	0.0	2.0	0.0
d ET (A) c	0.0	-2.0	0.0	-2.0	0.0	-2.0	0.0	-2.0	0.0	-2.0	0.0	-2.0

Major tetrachord	E _b -F-G-A _b	B _b -C-D-E _b	F-G-A-B _b	C-D-E-F	G-A-B-C	D-E-F _# -G	A-B-C _# -D	E-F _# -G _# -A	B-C _# -D _# -E	F _# -G _# -A _# -B	D _b -E _b -F-G _b	A _b -B _b -C-D _b
Ut-Re	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0
Re-Mi	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0
Mi-Fa	98.0	102.0	98.0	102.0	98.0	102.0	98.0	102.0	98.0	102.0	98.0	102.0
Minor tetrachord	E _b -F-G _b -A _b	B _b -C-D _b -E _b	F-G-A _b -B _b	C-D-E _b -F	G-A-B _b -C	D-E-F-G	A-B-C-D	E-F _# -G-A	B-C _# -D-E	F _# -G _# -A-B	C _# -D _# -E-F _#	G _# -A _# -B-C _#
Re-Mi	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0
Mi-Fa	98.0	102.0	98.0	102.0	98.0	102.0	98.0	102.0	98.0	102.0	98.0	102.0
Fa-Sol	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0

Handling diesis	A _b -C 7	D _b -F 7	F _# -A _# 7	B-D _# 7
within the octave	E-G _# 7	A-C _# 7	D-F _# 7	G-B 7
(portions of 21)	C-E 7	F-A 7	B _b -D 7	E _b -G 7

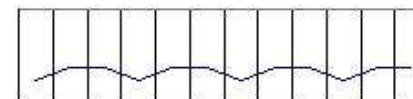
Johann Georg Neidhardt, 1732

Neidhardt (1732) 5th-circle #4

	E_b	B_b	F	C	G	D	A	E	B	F[#]	C[#]	G[#]
Fifth in TU	-180	0	0	-180	0	0	-180	0	0	-180	0	0
Fifth in PC	- 1/4			- 1/4			- 1/4			- 1/4		
Fifth in SC	- 3/11			- 3/11			- 3/11			- 3/11		
Maj3rd % SC	45.5%	72.7%	72.7%	45.5%	72.7%	72.7%	45.5%	72.7%	72.7%	45.5%	72.7%	72.7%
Diesis portion	23.8%	38.1%	38.1%	23.8%	38.1%	38.1%	23.8%	38.1%	38.1%	23.8%	38.1%	38.1%
Min3rd % SC	-72.7%	-72.7%	-72.7%	-72.7%	-72.7%	-72.7%	-72.7%	-72.7%	-72.7%	-72.7%	-72.7%	-72.7%
Tone c	198.0	203.9	198.0	198.0	203.9	198.0	198.0	203.9	198.0	198.0	203.9	198.0
Semitone c	96.1	102.0	102.0	96.1	102.0	102.0	96.1	102.0	102.0	96.1	102.0	102.0
Enharmonic	<i>D[#]</i>	<i>A[#]</i>	<i>E[#]</i>	<i>B[#]</i>	<i>F_x</i>	<i>C_x</i>	<i>B_{bb}</i>	<i>F_b</i>	<i>C_b</i>	<i>G_b</i>	<i>D_b</i>	<i>A_b</i>
Enharm err % PC	75.0%	66.7%	83.3%	100.0%	91.7%	108.3%	-75.0%	-83.3%	-66.7%	-50.0%	-58.3%	-41.7%
d 1/6PC (C) c	-5.9	-7.8	-3.9	0.0	-2.0	2.0	5.9	3.9	7.8	11.7	9.8	13.7
Error % PC	-25.0%	-33.3%	-16.7%	0.0%	-8.3%	8.3%	25.0%	16.7%	33.3%	50.0%	41.7%	58.3%
Primary function	E_b	B_b	F	C	G	D	A	E	B	F[#]	C[#]	G[#]
d 1/6SC (C) c	-4.9	-7.2	-3.6	0.0	-2.3	1.3	4.9	2.6	6.2	9.8	7.5	11.1
d ET (C) c	0.0	-3.9	-2.0	0.0	-3.9	-2.0	0.0	-3.9	-2.0	0.0	-3.9	-2.0
d ET (A) c	0.0	-3.9	-2.0	0.0	-3.9	-2.0	0.0	-3.9	-2.0	0.0	-3.9	-2.0

Major tetrachord	E _b -F-G-A _b	B _b -C-D-E _b	F-G-A-B _b	C-D-E-F	G-A-B-C	D-E-F [#] -G	A-B-C [#] -D	E-F [#] -G [#] -A	B-C [#] -D [#] -E	F [#] -G [#] -A [#] -B	D _b -E _b -F-G _b	A _b -B _b -C-D _b
Ut-Re	198.0	203.9	198.0	198.0	203.9	198.0	198.0	203.9	198.0	198.0	203.9	198.0
Re-Mi	198.0	198.0	203.9	198.0	198.0	203.9	198.0	198.0	203.9	198.0	198.0	203.9
Mi-Fa	102.0	102.0	96.1	102.0	102.0	96.1	102.0	102.0	96.1	102.0	102.0	96.1
Minor tetrachord	E _b -F-G _b -A _b	B _b -C-D _b -E _b	F-G-A _b -B _b	C-D-E _b -F	G-A-B _b -C	D-E-F-G	A-B-C-D	E-F [#] -G-A	B-C [#] -D-E	F [#] -G [#] -A-B	C [#] -D [#] -E-F [#]	G [#] -A [#] -B-C [#]
Re-Mi	198.0	203.9	198.0	198.0	203.9	198.0	198.0	203.9	198.0	198.0	203.9	198.0
Mi-Fa	102.0	96.1	102.0	102.0	96.1	102.0	102.0	96.1	102.0	102.0	96.1	102.0
Fa-Sol	198.0	203.9	198.0	198.0	203.9	198.0	198.0	203.9	198.0	198.0	203.9	198.0

Handling diesis	Ab-C 8	Db-F 8	F [#] -A [#] 5	B-D [#] 8
within the octave	E-G [#] 8	A-C [#] 5	D-F [#] 8	G-B 8
(portions of 21)	C-E 5	F-A 8	B _b -D 8	E _b -G 5



Johann Georg Neidhardt, 1732

Neidhardt (1732) 5th-circle #5

	E_b	B_b	F	C	G	D	A	E	B	F[#]	C[#]	G[#]
Fifth in TU	-60	-60	-120	-60	-60	-120	60	-60	-120	-60	-60	0
Fifth in PC	- 1/12	- 1/12	- 1/6	- 1/12	- 1/12	- 1/6	1/12	- 1/12	- 1/6	- 1/12	- 1/12	
Fifth in SC	- 1/11	- 1/11	- 2/11	- 1/11	- 1/11	- 2/11	1/11	- 1/11	- 2/11	- 1/11	- 1/11	
Maj3rd % SC	54.5%	54.5%	45.5%	72.7%	72.7%	63.6%	72.7%	54.5%	63.6%	72.7%	72.7%	63.6%
Diesis portion	28.6%	28.6%	23.8%	38.1%	38.1%	33.3%	38.1%	28.6%	33.3%	38.1%	38.1%	33.3%
Min3rd % SC	-81.8%	-81.8%	-81.8%	-63.6%	-63.6%	-63.6%	-63.6%	-81.8%	-81.8%	-81.8%	-63.6%	-63.6%
Tone c	200.0	198.0	198.0	200.0	198.0	202.0	203.9	198.0	198.0	200.0	202.0	202.0
Semitone c	100.0	100.0	98.0	100.0	100.0	102.0	103.9	100.0	98.0	100.0	100.0	98.0
Enharmonic	<i>D[#]</i>	<i>A[#]</i>	<i>E[#]</i>	<i>B[#]</i>	<i>F_x</i>	<i>C_x</i>	<i>B_{bb}</i>	<i>F_b</i>	<i>C_b</i>	<i>G_b</i>	<i>D_b</i>	<i>A_b</i>
Enharm err % PC	83.3%	91.7%	100.0%	100.0%	108.3%	116.7%	-83.3%	-58.3%	-50.0%	-50.0%	-41.7%	-33.3%
d 1/6PC (C) c	-3.9	-2.0	0.0	0.0	2.0	3.9	3.9	9.8	11.7	11.7	13.7	15.6
Error % PC	-16.7%	-8.3%	0.0%	0.0%	8.3%	16.7%	16.7%	41.7%	50.0%	50.0%	58.3%	66.7%
Primary function	E_b	B_b	F	C	G	D	A	E	B	F[#]	C[#]	G[#]
d 1/6SC (C) c	-2.9	-1.3	0.3	0.0	1.6	3.3	2.9	8.5	10.1	9.8	11.4	13.0
d ET (C) c	2.0	2.0	2.0	0.0	0.0	0.0	-2.0	2.0	2.0	0.0	0.0	0.0
d ET (A) c	3.9	3.9	3.9	2.0	2.0	2.0	0.0	3.9	3.9	2.0	2.0	2.0

Major tetrachord	E_b-F-G-A_b	B_b-C-D-E_b	F-G-A-B_b	C-D-E-F	G-A-B-C	D-E-F[#]-G	A-B-C[#]-D	E-F[#]-G[#]-A	B-C[#]-D[#]-E	F[#]-G[#]-A[#]-B	D_b-E_b-F-G_b	A_b-B_b-C-D_b
Ut-Re	200.0	198.0	198.0	200.0	198.0	202.0	203.9	198.0	198.0	200.0	202.0	202.0
Re-Mi	198.0	200.0	198.0	202.0	203.9	198.0	198.0	200.0	202.0	202.0	200.0	198.0
Mi-Fa	100.0	102.0	103.9	100.0	98.0	100.0	100.0	98.0	100.0	100.0	98.0	100.0
Minor tetrachord	E_b-F-G_b-A_b	B_b-C-D_b-E_b	F-G-A_b-B_b	C-D-E_b-F	G-A-B_b-C	D-E-F-G	A-B-C-D	E-F[#]-G-A	B-C[#]-D-E	F[#]-G[#]-A-B	C[#]-D[#]-E-F[#]	G[#]-A[#]-B-C[#]
Re-Mi	200.0	198.0	198.0	200.0	198.0	202.0	203.9	198.0	198.0	200.0	202.0	202.0
Mi-Fa	98.0	100.0	100.0	102.0	103.9	100.0	98.0	100.0	100.0	98.0	100.0	100.0
Fa-Sol	200.0	202.0	202.0	200.0	198.0	198.0	200.0	198.0	202.0	203.9	198.0	198.0

Handling diesis	Ab-C 7	Db-F 8	F[#]-A[#] 8	B-D[#] 7
within the octave	E-G[#] 6	A-C[#] 8	D-F[#] 7	G-B 8
(portions of 21)	C-E 8	F-A 5	B_b-D 6	E_b-G 6



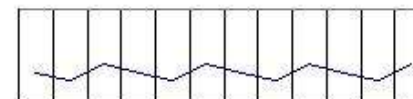
Johann Georg Neidhardt, 1732

Neidhardt (1732) 5th-circle #6

	E_b	B_b	F	C	G	D	A	E	B	F[#]	C[#]	G[#]
Fifth in TU	-60	-180	60	-60	-180	60	-60	-180	60	-60	-180	60
Fifth in PC	- 1/12	- 1/4	1/12	- 1/12	- 1/4	1/12	- 1/12	- 1/4	1/12	- 1/12	- 1/4	1/12
Fifth in SC	- 1/11	- 3/11	1/11	- 1/11	- 3/11	1/11	- 1/11	- 3/11	1/11	- 1/11	- 3/11	1/11
Maj3rd % SC	63.6%	45.5%	81.8%	63.6%	45.5%	81.8%	63.6%	45.5%	81.8%	63.6%	45.5%	81.8%
Diesis portion	33.3%	23.8%	42.9%	33.3%	23.8%	42.9%	33.3%	23.8%	42.9%	33.3%	23.8%	42.9%
Min3rd % SC	-72.7%	-72.7%	-72.7%	-72.7%	-72.7%	-72.7%	-72.7%	-72.7%	-72.7%	-72.7%	-72.7%	-72.7%
Tone c	196.1	200.0	203.9	196.1	200.0	203.9	196.1	200.0	203.9	196.1	200.0	203.9
Semitone c	100.0	96.1	103.9	100.0	96.1	103.9	100.0	96.1	103.9	100.0	96.1	103.9
Enharmonic	<i>D[#]</i>	<i>A[#]</i>	<i>E[#]</i>	<i>B[#]</i>	<i>F_x</i>	<i>C_x</i>	<i>B_{bb}</i>	<i>F_b</i>	<i>C_b</i>	<i>G_b</i>	<i>D_b</i>	<i>A_b</i>
Enharm err % PC	75.0%	83.3%	75.0%	100.0%	108.3%	100.0%	-75.0%	-66.7%	-75.0%	-50.0%	-41.7%	-50.0%
d 1/6PC (C) c	-5.9	-3.9	-5.9	0.0	2.0	0.0	5.9	7.8	5.9	11.7	13.7	11.7
Error % PC	-25.0%	-16.7%	-25.0%	0.0%	8.3%	0.0%	25.0%	33.3%	25.0%	50.0%	58.3%	50.0%
Primary function	E_b	B_b	F	C	G	D	A	E	B	F[#]	C[#]	G[#]
d 1/6SC (C) c	-4.9	-3.3	-5.5	0.0	1.6	-0.7	4.9	6.5	4.2	9.8	11.4	9.1
d ET (C) c	0.0	0.0	-3.9	0.0	0.0	-3.9	0.0	0.0	-3.9	0.0	0.0	-3.9
d ET (A) c	0.0	0.0	-3.9	0.0	0.0	-3.9	0.0	0.0	-3.9	0.0	0.0	-3.9

Major tetrachord	E _b -F-G-A _b	B _b -C-D-E _b	F-G-A-B _b	C-D-E-F	G-A-B-C	D-E-F [#] -G	A-B-C [#] -D	E-F [#] -G [#] -A	B-C [#] -D [#] -E	F [#] -G [#] -A [#] -B	D _b -E _b -F-G _b	A _b -B _b -C-D _b
Ut-Re	196.1	200.0	203.9	196.1	200.0	203.9	196.1	200.0	203.9	196.1	200.0	203.9
Re-Mi	203.9	196.1	200.0	203.9	196.1	200.0	203.9	196.1	200.0	203.9	196.1	200.0
Mi-Fa	96.1	103.9	100.0	96.1	103.9	100.0	96.1	103.9	100.0	96.1	103.9	100.0
Minor tetrachord	E _b -F-G _b -A _b	B _b -C-D _b -E _b	F-G-A _b -B _b	C-D-E _b -F	G-A-B _b -C	D-E-F-G	A-B-C-D	E-F [#] -G-A	B-C [#] -D-E	F [#] -G [#] -A-B	C [#] -D [#] -E-F [#]	G [#] -A [#] -B-C [#]
Re-Mi	196.1	200.0	203.9	196.1	200.0	203.9	196.1	200.0	203.9	196.1	200.0	203.9
Mi-Fa	103.9	100.0	96.1	103.9	100.0	96.1	103.9	100.0	96.1	103.9	100.0	96.1
Fa-Sol	196.1	200.0	203.9	196.1	200.0	203.9	196.1	200.0	203.9	196.1	200.0	203.9

Handling diesis	A _b -C 9	D _b -F 5	F [#] -A [#] 7	B-D [#] 9
within the octave	E-G [#] 5	A-C [#] 7	D-F [#] 9	G-B 5
(portions of 21)	C-E 7	F-A 9	B _b -D 5	E _b -G 7



Johann Georg Neidhardt, 1732

Neidhardt (1732) 5th-circle #7												
	E_b	B_b	F	C	G	D	A	E	B	F[#]	C[#]	G[#]
Fifth in TU	-120	-120	120	-180	-120	-120	120	-120	-120	-120	120	-60
Fifth in PC	- 1/6	- 1/6	1/6	- 1/4	- 1/6	- 1/6	1/6	- 1/6	- 1/6	- 1/6	1/6	- 1/12
Fifth in SC	- 2/11	- 2/11	2/11	- 3/11	- 2/11	- 2/11	2/11	- 2/11	- 2/11	- 2/11	2/11	- 1/11
Maj3rd % SC	54.5%	54.5%	54.5%	54.5%	63.6%	63.6%	63.6%	63.6%	72.7%	72.7%	72.7%	72.7%
Diesis portion	28.6%	28.6%	28.6%	28.6%	33.3%	33.3%	33.3%	33.3%	38.1%	38.1%	38.1%	38.1%
Min3rd % SC	-90.9%	-90.9%	-54.5%	-81.8%	-72.7%	-72.7%	-36.4%	-81.8%	-81.8%	-81.8%	-45.5%	-81.8%
Tone c	196.1	203.9	202.0	194.1	196.1	203.9	203.9	196.1	196.1	203.9	205.9	198.0
Semitone c	100.0	100.0	100.0	92.2	102.0	103.9	103.9	96.1	103.9	102.0	102.0	94.1
Enharmonic	<i>D[#]</i>	<i>A[#]</i>	<i>E[#]</i>	<i>B[#]</i>	<i>F_x</i>	<i>C_x</i>	<i>B_{bb}</i>	<i>F_b</i>	<i>C_b</i>	<i>G_b</i>	<i>D_b</i>	<i>A_b</i>
Enharm err % PC	66.7%	66.7%	66.7%	100.0%	91.7%	91.7%	-108.3%	-75.0%	-75.0%	-75.0%	-75.0%	-41.7%
d 1/6PC (C) c	-7.8	-7.8	-7.8	0.0	-2.0	-2.0	-2.0	5.9	5.9	5.9	5.9	13.7
Error % PC	-33.3%	-33.3%	-33.3%	0.0%	-8.3%	-8.3%	-8.3%	25.0%	25.0%	25.0%	25.0%	58.3%
Primary function	E_b	B_b	F	C	G	D	A	E	B	F[#]	C[#]	G[#]
d 1/6SC (C) c	-6.8	-7.2	-7.5	0.0	-2.3	-2.6	-2.9	4.6	4.2	3.9	3.6	11.1
d ET (C) c	-2.0	-3.9	-5.9	0.0	-3.9	-5.9	-7.8	-2.0	-3.9	-5.9	-7.8	-2.0
d ET (A) c	5.9	3.9	2.0	7.8	3.9	2.0	0.0	5.9	3.9	2.0	0.0	5.9

Major tetrachord	E _b -F-G-A _b	B _b -C-D-E _b	F-G-A-B _b	C-D-E-F	G-A-B-C	D-E-F [#] -G	A-B-C [#] -D	E-F [#] -G [#] -A	B-C [#] -D [#] -E	F [#] -G [#] -A [#] -B	D _b -E _b -F-G _b	A _b -B _b -C-D _b
Ut-Re	196.1	203.9	202.0	194.1	196.1	203.9	203.9	196.1	196.1	203.9	205.9	198.0
Re-Mi	202.0	194.1	196.1	203.9	203.9	196.1	196.1	203.9	205.9	198.0	196.1	203.9
Mi-Fa	102.0	103.9	103.9	96.1	103.9	102.0	102.0	94.1	100.0	100.0	100.0	92.2
Minor tetrachord	E _b -F-G _b -A _b	B _b -C-D _b -E _b	F-G-A _b -B _b	C-D-E _b -F	G-A-B _b -C	D-E-F-G	A-B-C-D	E-F [#] -G-A	B-C [#] -D-E	F [#] -G [#] -A-B	C [#] -D [#] -E-F [#]	G [#] -A [#] -B-C [#]
Re-Mi	196.1	203.9	202.0	194.1	196.1	203.9	203.9	196.1	196.1	203.9	205.9	198.0
Mi-Fa	100.0	92.2	102.0	103.9	103.9	96.1	103.9	102.0	102.0	94.1	100.0	100.0
Fa-Sol	203.9	205.9	198.0	196.1	203.9	202.0	194.1	196.1	203.9	203.9	196.1	196.1


Handling diesis within the octave (portions of 21)	Ab-C 8 E-G [#] 7 C-E 6	Db-F 8 A-C [#] 7 F-A 6	F [#] -A [#] 8 D-F [#] 7 B _b -D 6	B-D [#] 8 G-B 7 E _b -G 6
--	--	--	--	---

Johann Georg Neidhardt, 1732

Neidhardt (1732) 5th-circle #8 (Big City)												
	E_b	B_b	F	C	G	D	A	E	B	F[#]	C[#]	G[#]
Fifth in TU	0	-60	-60	-120	-120	-120	-60	0	-60	-60	-60	0
Fifth in PC		- 1/12	- 1/12	- 1/6	- 1/6	- 1/6	- 1/12		- 1/12	- 1/12	- 1/12	
Fifth in SC		- 1/11	- 1/11	- 2/11	- 2/11	- 2/11	- 1/11		- 1/11	- 1/11	- 1/11	
Maj3rd % SC	63.6%	45.5%	36.4%	36.4%	54.5%	63.6%	72.7%	72.7%	72.7%	81.8%	81.8%	81.8%
Diesis portion	33.3%	23.8%	19.0%	19.0%	28.6%	33.3%	38.1%	38.1%	38.1%	42.9%	42.9%	42.9%
Min3rd % SC	-81.8%	-90.9%	-90.9%	-81.8%	-63.6%	-54.5%	-45.5%	-54.5%	-72.7%	-81.8%	-81.8%	-72.7%
Tone c	202.0	200.0	198.0	196.1	196.1	198.0	202.0	202.0	200.0	200.0	202.0	203.9
Semitone c	96.1	96.1	96.1	96.1	98.0	102.0	105.9	105.9	103.9	102.0	100.0	98.0
Enharmonic	<i>D[#]</i>	<i>A[#]</i>	<i>E[#]</i>	<i>B[#]</i>	<i>F_x</i>	<i>C_x</i>	<i>B_{bb}</i>	<i>F_b</i>	<i>C_b</i>	<i>G_b</i>	<i>D_b</i>	<i>A_b</i>
Enharm err % PC	66.7%	83.3%	91.7%	100.0%	100.0%	100.0%	-100.0%	-91.7%	-75.0%	-66.7%	-58.3%	-50.0%
d 1/6PC (C) c	-7.8	-3.9	-2.0	0.0	0.0	0.0	0.0	2.0	5.9	7.8	9.8	11.7
Error % PC	-33.3%	-16.7%	-8.3%	0.0%	0.0%	0.0%	0.0%	8.3%	25.0%	33.3%	41.7%	50.0%
	E_b	B_b	F	C	G	D	A	E	B	F[#]	C[#]	G[#]
d 1/6SC (C) c	-6.8	-3.3	-1.6	0.0	-0.3	-0.7	-1.0	0.7	4.2	5.9	7.5	9.1
d ET (C) c	-2.0	0.0	0.0	0.0	-2.0	-3.9	-5.9	-5.9	-3.9	-3.9	-3.9	-3.9
d ET (A) c	3.9	5.9	5.9	5.9	3.9	2.0	0.0	0.0	2.0	2.0	2.0	2.0

Major tetrachord	E _b -F-G-A _b	B _b -C-D-E _b	F-G-A-B _b	C-D-E-F	G-A-B-C	D-E-F [#] -G	A-B-C [#] -D	E-F [#] -G [#] -A	B-C [#] -D [#] -E	F [#] -G [#] -A [#] -B	D _b -E _b -F-G _b	A _b -B _b -C-D _b
Ut-Re	202.0	200.0	198.0	196.1	196.1	198.0	202.0	202.0	200.0	200.0	202.0	203.9
Re-Mi	198.0	196.1	196.1	198.0	202.0	202.0	200.0	200.0	202.0	203.9	202.0	200.0
Mi-Fa	98.0	102.0	105.9	105.9	103.9	102.0	100.0	98.0	96.1	96.1	96.1	96.1
Minor tetrachord	E _b -F-G _b -A _b	B _b -C-D _b -E _b	F-G-A _b -B _b	C-D-E _b -F	G-A-B _b -C	D-E-F-G	A-B-C-D	E-F [#] -G-A	B-C [#] -D-E	F [#] -G [#] -A-B	C [#] -D [#] -E-F [#]	G [#] -A [#] -B-C [#]
Re-Mi	202.0	200.0	198.0	196.1	196.1	198.0	202.0	202.0	200.0	200.0	202.0	203.9
Mi-Fa	96.1	96.1	98.0	102.0	105.9	105.9	103.9	102.0	100.0	98.0	96.1	96.1
Fa-Sol	200.0	202.0	203.9	202.0	200.0	198.0	196.1	196.1	198.0	202.0	202.0	200.0

Handling diesis	A _b -C 9	D _b -F 9	F [#] -A [#] 9	B-D [#] 8
within the octave	E-G [#] 8	A-C [#] 8	D-F [#] 7	G-B 6
(portions of 21)	C-E 4	F-A 4	B _b -D 5	E _b -G 7



Johann Georg Neidhardt, 1732

Neidhardt (1732) 5th-circle #9												
	E_b	B_b	F	C	G	D	A	E	B	F_#	C_#	G_#
Fifth in TU	-180	0	0	-60	-180	0	0	-60	-180	0	0	-60
Fifth in PC	- 1/4			- 1/12	- 1/4			- 1/12	- 1/4			- 1/12
Fifth in SC	- 3/11			- 1/11	- 3/11			- 1/11	- 3/11			- 1/11
Maj3rd % SC	63.6%	63.6%	63.6%	63.6%	63.6%	63.6%	63.6%	63.6%	63.6%	63.6%	63.6%	63.6%
Diesis portion	33.3%	33.3%	33.3%	33.3%	33.3%	33.3%	33.3%	33.3%	33.3%	33.3%	33.3%	33.3%
Min3rd % SC	-90.9%	-63.6%	-63.6%	-72.7%	-90.9%	-63.6%	-63.6%	-72.7%	-90.9%	-63.6%	-63.6%	-72.7%
Tone c	198.0	203.9	202.0	196.1	198.0	203.9	202.0	196.1	198.0	203.9	202.0	196.1
Semitone c	100.0	103.9	98.0	98.0	100.0	103.9	98.0	98.0	100.0	103.9	98.0	98.0
Enharmonic	<i>D_#</i>	<i>A_#</i>	<i>E_#</i>	<i>B_#</i>	<i>F_x</i>	<i>C_x</i>	<i>B_{bb}</i>	<i>F_b</i>	<i>C_b</i>	<i>G_b</i>	<i>D_b</i>	<i>A_b</i>
Enharm err % PC	75.0%	66.7%	83.3%	100.0%	108.3%	100.0%	-83.3%	-66.7%	-58.3%	-66.7%	-50.0%	-33.3%
d 1/6PC (C) c	-5.9	-7.8	-3.9	0.0	2.0	0.0	3.9	7.8	9.8	7.8	11.7	15.6
Error % PC	-25.0%	-33.3%	-16.7%	0.0%	8.3%	0.0%	16.7%	33.3%	41.7%	33.3%	50.0%	66.7%
	E_b	B_b	F	C	G	D	A	E	B	F_#	C_#	G_#
d 1/6SC (C) c	-4.9	-7.2	-3.6	0.0	1.6	-0.7	2.9	6.5	8.1	5.9	9.5	13.0
d ET (C) c	0.0	-3.9	-2.0	0.0	0.0	-3.9	-2.0	0.0	0.0	-3.9	-2.0	0.0
d ET (A) c	2.0	-2.0	0.0	2.0	2.0	-2.0	0.0	2.0	2.0	-2.0	0.0	2.0

Major tetrachord	E _b -F-G-A _b	B _b -C-D-E _b	F-G-A-B _b	C-D-E-F	G-A-B-C	D-E-F _# -G	A-B-C _# -D	E-F _# -G _# -A	B-C _# -D _# -E	F _# -G _# -A _# -B	D _b -E _b -F-G _b	A _b -B _b -C-D _b
Ut-Re	198.0	203.9	202.0	196.1	198.0	203.9	202.0	196.1	198.0	203.9	202.0	196.1
Re-Mi	202.0	196.1	198.0	203.9	202.0	196.1	198.0	203.9	202.0	196.1	198.0	203.9
Mi-Fa	100.0	103.9	98.0	98.0	100.0	103.9	98.0	98.0	100.0	103.9	98.0	98.0
Minor tetrachord	E _b -F-G _b -A _b	B _b -C-D _b -E _b	F-G-A _b -B _b	C-D-E _b -F	G-A-B _b -C	D-E-F-G	A-B-C-D	E-F _# -G-A	B-C _# -D-E	F _# -G _# -A-B	C _# -D _# -E-F _#	G _# -A _# -B-C _#
Re-Mi	198.0	203.9	202.0	196.1	198.0	203.9	202.0	196.1	198.0	203.9	202.0	196.1
Mi-Fa	98.0	98.0	100.0	103.9	98.0	98.0	100.0	103.9	98.0	98.0	100.0	103.9
Fa-Sol	203.9	202.0	196.1	198.0	203.9	202.0	196.1	198.0	203.9	202.0	196.1	198.0

Handling diesis	A _b -C 7	D _b -F 7	F _# -A _# 7	B-D _# 7
within the octave	E-G _# 7	A-C _# 7	D-F _# 7	G-B 7
(portions of 21)	C-E 7	F-A 7	B _b -D 7	E _b -G 7

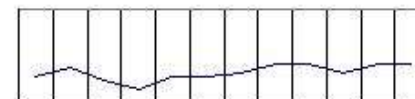
Johann Georg Neidhardt, 1732

Neidhardt (1732) 5th-circle #10

	E_b	B_b	F	C	G	D	A	E	B	F[#]	C[#]	G[#]
Fifth in TU	-120	0	0	-180	0	-180	-120	0	0	-120	0	0
Fifth in PC	- 1/6			- 1/4		- 1/4	- 1/6			- 1/6		
Fifth in SC	- 2/11			- 3/11		- 3/11	- 2/11			- 2/11		
Maj3rd % SC	54.5%	72.7%	45.5%	27.3%	54.5%	54.5%	63.6%	81.8%	81.8%	63.6%	81.8%	81.8%
Diesis portion	28.6%	38.1%	23.8%	14.3%	28.6%	28.6%	33.3%	42.9%	42.9%	33.3%	42.9%	42.9%
Min3rd % SC	-81.8%	-81.8%	-81.8%	-81.8%	-72.7%	-72.7%	-45.5%	-54.5%	-54.5%	-81.8%	-81.8%	-81.8%
Tone c	200.0	203.9	198.0	198.0	198.0	194.1	200.0	203.9	200.0	200.0	203.9	200.0
Semitone c	94.1	98.0	98.0	94.1	100.0	100.0	102.0	105.9	105.9	100.0	103.9	98.0
Enharmonic	<i>D[#]</i>	<i>A[#]</i>	<i>E[#]</i>	<i>B[#]</i>	<i>F_x</i>	<i>C_x</i>	<i>B_{bb}</i>	<i>F_b</i>	<i>C_b</i>	<i>G_b</i>	<i>D_b</i>	<i>A_b</i>
Enharm err % PC	66.7%	66.7%	83.3%	100.0%	91.7%	108.3%	-100.0%	-100.0%	-83.3%	-66.7%	-66.7%	-50.0%
d 1/6PC (C) c	-7.8	-7.8	-3.9	0.0	-2.0	2.0	0.0	0.0	3.9	7.8	7.8	11.7
Error % PC	-33.3%	-33.3%	-16.7%	0.0%	-8.3%	8.3%	0.0%	0.0%	16.7%	33.3%	33.3%	50.0%
Primary function	E_b	B_b	F	C	G	D	A	E	B	F[#]	C[#]	G[#]
d 1/6SC (C) c	-6.8	-7.2	-3.6	0.0	-2.3	1.3	-1.0	-1.3	2.3	5.9	5.5	9.1
d ET (C) c	-2.0	-3.9	-2.0	0.0	-3.9	-2.0	-5.9	-7.8	-5.9	-3.9	-5.9	-3.9
d ET (A) c	3.9	2.0	3.9	5.9	2.0	3.9	0.0	-2.0	0.0	2.0	0.0	2.0

Major tetrachord	E_b-F-G-A_b	B_b-C-D-E_b	F-G-A-B_b	C-D-E-F	G-A-B-C	D-E-F[#]-G	A-B-C[#]-D	E-F[#]-G[#]-A	B-C[#]-D[#]-E	F[#]-G[#]-A[#]-B	D_b-E_b-F-G_b	A_b-B_b-C-D_b
Ut-Re	200.0	203.9	198.0	198.0	198.0	194.1	200.0	203.9	200.0	200.0	203.9	200.0
Re-Mi	198.0	198.0	198.0	194.1	200.0	203.9	200.0	200.0	203.9	200.0	200.0	203.9
Mi-Fa	100.0	100.0	102.0	105.9	105.9	100.0	103.9	98.0	94.1	98.0	98.0	94.1
Minor tetrachord	E_b-F-G_b-A_b	B_b-C-D_b-E_b	F-G-A_b-B_b	C-D-E_b-F	G-A-B_b-C	D-E-F-G	A-B-C-D	E-F[#]-G-A	B-C[#]-D-E	F[#]-G[#]-A-B	C[#]-D[#]-E-F[#]	G[#]-A[#]-B-C[#]
Re-Mi	200.0	203.9	198.0	198.0	198.0	194.1	200.0	203.9	200.0	200.0	203.9	200.0
Mi-Fa	98.0	94.1	100.0	100.0	102.0	105.9	105.9	100.0	103.9	98.0	94.1	98.0
Fa-Sol	200.0	203.9	200.0	200.0	203.9	198.0	198.0	198.0	194.1	200.0	203.9	200.0

Handling diesis	A_b-C 9	D_b-F 9	F[#]-A[#] 7	B-D[#] 9
within the octave	E-G[#] 9	A-C[#] 7	D-F[#] 6	G-B 6
(portions of 21)	C-E 3	F-A 5	B_b-D 8	E_b-G 6



Johann Georg Neidhardt, 1732

Neidhardt (1732) 5th-circle #11

	E_b	B_b	F	C	G	D	A	E	B	F[#]	C[#]	G[#]
Fifth in TU	-60	60	-60	-60	-120	-180	-60	60	-60	-120	60	-180
Fifth in PC	- 1/12	1/12	- 1/12	- 1/12	- 1/6	- 1/4	- 1/12	1/12	- 1/12	- 1/6	1/12	- 1/4
Fifth in SC	- 1/11	1/11	- 1/11	- 1/11	- 2/11	- 3/11	- 1/11	1/11	- 1/11	- 2/11	1/11	- 3/11
Maj3rd % SC	81.8%	72.7%	36.4%	36.4%	54.5%	63.6%	72.7%	90.9%	54.5%	54.5%	81.8%	63.6%
Diesis portion	42.9%	38.1%	19.0%	19.0%	28.6%	33.3%	38.1%	47.6%	28.6%	28.6%	42.9%	33.3%
Min3rd % SC	-63.6%	-72.7%	-72.7%	-90.9%	-90.9%	-63.6%	-45.5%	-45.5%	-72.7%	-90.9%	-81.8%	-81.8%
Tone c	203.9	203.9	200.0	198.0	194.1	196.1	203.9	203.9	198.0	202.0	200.0	196.1
Semitone c	98.0	102.0	98.0	96.1	100.0	98.0	102.0	105.9	102.0	102.0	102.0	94.1
Enharmonic	<i>D[#]</i>	<i>A[#]</i>	<i>E[#]</i>	<i>B[#]</i>	<i>F_x</i>	<i>C_x</i>	<i>B_{bb}</i>	<i>F_b</i>	<i>C_b</i>	<i>G_b</i>	<i>D_b</i>	<i>A_b</i>
Enharm err % PC	58.3%	66.7%	91.7%	100.0%	108.3%	108.3%	-100.0%	-91.7%	-66.7%	-58.3%	-58.3%	-33.3%
d 1/6PC (C) c	-9.8	-7.8	-2.0	0.0	2.0	2.0	0.0	2.0	7.8	9.8	9.8	15.6
Error % PC	-41.7%	-33.3%	-8.3%	0.0%	8.3%	8.3%	0.0%	8.3%	33.3%	41.7%	41.7%	66.7%
Primary function	E_b	B_b	F	C	G	D	A	E	B	F[#]	C[#]	G[#]
d 1/6SC (C) c	-8.8	-7.2	-1.6	0.0	1.6	1.3	-1.0	0.7	6.2	7.8	7.5	13.0
d ET (C) c	-3.9	-3.9	0.0	0.0	0.0	-2.0	-5.9	-5.9	-2.0	-2.0	-3.9	0.0
d ET (A) c	2.0	2.0	5.9	5.9	5.9	3.9	0.0	0.0	3.9	3.9	2.0	5.9

Major tetrachord	E _b -F-G-A _b	B _b -C-D-E _b	F-G-A-B _b	C-D-E-F	G-A-B-C	D-E-F [#] -G	A-B-C [#] -D	E-F [#] -G [#] -A	B-C [#] -D [#] -E	F [#] -G [#] -A [#] -B	D _b -E _b -F-G _b	A _b -B _b -C-D _b
Ut-Re	203.9	203.9	200.0	198.0	194.1	196.1	203.9	203.9	198.0	202.0	200.0	196.1
Re-Mi	200.0	198.0	194.1	196.1	203.9	203.9	198.0	202.0	200.0	196.1	203.9	203.9
Mi-Fa	100.0	98.0	102.0	105.9	102.0	102.0	102.0	94.1	98.0	102.0	98.0	96.1
Minor tetrachord	E _b -F-G _b -A _b	B _b -C-D _b -E _b	F-G-A _b -B _b	C-D-E _b -F	G-A-B _b -C	D-E-F-G	A-B-C-D	E-F [#] -G-A	B-C [#] -D-E	F [#] -G [#] -A-B	C [#] -D [#] -E-F [#]	G [#] -A [#] -B-C [#]
Re-Mi	203.9	203.9	200.0	198.0	194.1	196.1	203.9	203.9	198.0	202.0	200.0	196.1
Mi-Fa	98.0	96.1	100.0	98.0	102.0	105.9	102.0	102.0	102.0	94.1	98.0	102.0
Fa-Sol	202.0	200.0	196.1	203.9	203.9	200.0	198.0	194.1	196.1	203.9	203.9	198.0

Handling diesis	A _b -C 7	D _b -F 9	F [#] -A [#] 6	B-D [#] 6
within the octave	E-G [#] 10	A-C [#] 8	D-F [#] 7	G-B 6
(portions of 21)	C-E 4	F-A 4	B _b -D 8	E _b -G 9



Johann Georg Neidhardt, 1732

Neidhardt (1732) 5th-circle #12

	E_b	B_b	F	C	G	D	A	E	B	F_#	C_#	G_#
Fifth in TU	-180	0	0	-60	-120	0	-180	0	0	-60	-120	0
Fifth in PC	- 1/4			- 1/12	- 1/6		- 1/4			- 1/12	- 1/6	
Fifth in SC	- 3/11			- 1/11	- 2/11		- 3/11			- 1/11	- 2/11	
Maj3rd % SC	63.6%	72.7%	72.7%	45.5%	54.5%	72.7%	63.6%	72.7%	72.7%	45.5%	54.5%	72.7%
Diesis portion	33.3%	38.1%	38.1%	23.8%	28.6%	38.1%	33.3%	38.1%	38.1%	23.8%	28.6%	38.1%
Min3rd % SC	-72.7%	-54.5%	-72.7%	-72.7%	-90.9%	-72.7%	-72.7%	-54.5%	-72.7%	-72.7%	-90.9%	-72.7%
Tone c	198.0	203.9	202.0	198.0	200.0	198.0	198.0	203.9	202.0	198.0	200.0	198.0
Semitone c	96.1	102.0	102.0	100.0	98.0	102.0	96.1	102.0	102.0	100.0	98.0	102.0
Enharmonic	<i>D_#</i>	<i>A_#</i>	<i>E_#</i>	<i>B_#</i>	<i>F_x</i>	<i>C_x</i>	<i>B_{bb}</i>	<i>F_b</i>	<i>C_b</i>	<i>G_b</i>	<i>D_b</i>	<i>A_b</i>
Enharm err % PC	75.0%	66.7%	83.3%	100.0%	108.3%	108.3%	-75.0%	-83.3%	-66.7%	-50.0%	-41.7%	-41.7%
d 1/6PC (C) c	-5.9	-7.8	-3.9	0.0	2.0	2.0	5.9	3.9	7.8	11.7	13.7	13.7
Error % PC	-25.0%	-33.3%	-16.7%	0.0%	8.3%	8.3%	25.0%	16.7%	33.3%	50.0%	58.3%	58.3%
Primary function	E_b	B_b	F	C	G	D	A	E	B	F_#	C_#	G_#
d 1/6SC (C) c	-4.9	-7.2	-3.6	0.0	1.6	1.3	4.9	2.6	6.2	9.8	11.4	11.1
d ET (C) c	0.0	-3.9	-2.0	0.0	0.0	-2.0	0.0	-3.9	-2.0	0.0	0.0	-2.0
d ET (A) c	0.0	-3.9	-2.0	0.0	0.0	-2.0	0.0	-3.9	-2.0	0.0	0.0	-2.0

Major tetrachord	E _b -F-G-A _b	B _b -C-D-E _b	F-G-A-B _b	C-D-E-F	G-A-B-C	D-E-F _# -G	A-B-C _# -D	E-F _# -G _# -A	B-C _# -D _# -E	F _# -G _# -A _# -B	D _b -E _b -F-G _b	A _b -B _b -C-D _b
Ut-Re	198.0	203.9	202.0	198.0	200.0	198.0	198.0	203.9	202.0	198.0	200.0	198.0
Re-Mi	202.0	198.0	200.0	198.0	198.0	203.9	202.0	198.0	200.0	198.0	198.0	203.9
Mi-Fa	98.0	102.0	96.1	102.0	102.0	100.0	98.0	102.0	96.1	102.0	102.0	100.0
Minor tetrachord	E _b -F-G _b -A _b	B _b -C-D _b -E _b	F-G-A _b -B _b	C-D-E _b -F	G-A-B _b -C	D-E-F-G	A-B-C-D	E-F _# -G-A	B-C _# -D-E	F _# -G _# -A-B	C _# -D _# -E-F _#	G _# -A _# -B-C _#
Re-Mi	198.0	203.9	202.0	198.0	200.0	198.0	198.0	203.9	202.0	198.0	200.0	198.0
Mi-Fa	102.0	100.0	98.0	102.0	96.1	102.0	102.0	100.0	98.0	102.0	96.1	102.0
Fa-Sol	198.0	200.0	198.0	198.0	203.9	202.0	198.0	200.0	198.0	198.0	203.9	202.0

Handling diesis within the octave (portions of 21)	Ab-C 8	Db-F 6	F _# -A _# 5	B-D _# 8
	E-G _# 8	A-C _# 7	D-F _# 8	G-B 6
	C-E 5	F-A 8	B _b -D 8	E _b -G 7



Johann Georg Neidhardt, 1732

Chapter VII is a discussion showing how one may apportion the comma to favor the arrangements of 3^{rds}, as opposed to the previous focus on 5^{ths}. He begins with three example temperaments showing how the major and minor 3^{rds} are affected by comma shifts, or portions thereof:

Neidhardt (1732) Example #1 for chapter VII (before 3rd-circles)												
	E_b	B_b	F	C	G	D	A	E	B	F[#]	C[#]	G[#]
Fifth in TU	0	0	-720	0	0	0	-720	0	0	0	720	0
Fifth in PC			- 1/1				- 1/1				1/1	
Fifth in SC			- 12/11				- 12/11				12/11	
Maj3rd % SC	-9.1%	-9.1%	-9.1%	-9.1%	-9.1%	-9.1%	-9.1%	209.1%	209.1%	209.1%	209.1%	-9.1%
Diesis portion	-4.8%	-4.8%	-4.8%	-4.8%	-4.8%	-4.8%	-4.8%	109.5%	109.5%	109.5%	109.5%	-4.8%
Min3rd % SC	-209.1%	-209.1%	-100.0%	9.1%	9.1%	9.1%	-100.0%	9.1%	9.1%	9.1%	-100.0%	-209.1%
Tone c	203.9	180.4	180.4	203.9	203.9	180.4	180.4	203.9	203.9	227.4	227.4	203.9
Semitone c	66.8	66.8	66.8	90.2	113.7	113.7	113.7	137.1	113.7	113.7	113.7	90.2
Enharmonic	<i>D[#]</i>	<i>A[#]</i>	<i>E[#]</i>	<i>B[#]</i>	<i>F_x</i>	<i>C_x</i>	<i>B_{bb}</i>	<i>F_b</i>	<i>C_b</i>	<i>G_b</i>	<i>D_b</i>	<i>A_b</i>
Enharm err % PC	150.0%	166.7%	183.3%	100.0%	116.7%	133.3%	-50.0%	-133.3%	-116.7%	-100.0%	-83.3%	33.3%
d 1/6PC (C) c	11.7	15.6	19.6	0.0	3.9	7.8	11.7	-7.8	-3.9	0.0	3.9	31.3
Error % PC	50.0%	66.7%	83.3%	0.0%	16.7%	33.3%	50.0%	-33.3%	-16.7%	0.0%	16.7%	133.3%
	E_b	B_b	F	C	G	D	A	E	B	F[#]	C[#]	G[#]
d 1/6SC (C) c	12.7	16.3	19.9	0.0	3.6	7.2	10.8	-9.1	-5.5	-2.0	1.6	28.7
d ET (C) c	17.6	19.6	21.5	0.0	2.0	3.9	5.9	-15.6	-13.7	-11.7	-9.8	15.6
d ET (A) c	11.7	13.7	15.6	-5.9	-3.9	-2.0	0.0	-21.5	-19.6	-17.6	-15.6	9.8

Major tetrachord	E _b -F-G-A _b	B _b -C-D-E _b	F-G-A-B _b	C-D-E-F	G-A-B-C	D-E-F [#] -G	A-B-C [#] -D	E-F [#] -G [#] -A	B-C [#] -D [#] -E	F [#] -G [#] -A [#] -B	D _b -E _b -F-G _b	A _b -B _b -C-D _b
Ut-Re	203.9	180.4	180.4	203.9	203.9	180.4	180.4	203.9	203.9	227.4	227.4	203.9
Re-Mi	180.4	203.9	203.9	180.4	180.4	203.9	203.9	227.4	227.4	203.9	203.9	180.4
Mi-Fa	113.7	113.7	113.7	137.1	113.7	113.7	113.7	90.2	66.8	66.8	66.8	90.2
Minor tetrachord	E _b -F-G _b -A _b	B _b -C-D _b -E _b	F-G-A _b -B _b	C-D-E _b -F	G-A-B _b -C	D-E-F-G	A-B-C-D	E-F [#] -G-A	B-C [#] -D-E	F [#] -G [#] -A-B	C [#] -D [#] -E-F [#]	G [#] -A [#] -B-C [#]
Re-Mi	203.9	180.4	180.4	203.9	203.9	180.4	180.4	203.9	203.9	227.4	227.4	203.9
Mi-Fa	66.8	90.2	113.7	113.7	113.7	137.1	113.7	113.7	113.7	90.2	66.8	66.8
Fa-Sol	227.4	227.4	203.9	203.9	180.4	180.4	203.9	203.9	180.4	180.4	203.9	203.9

Handling diesis within the octave (portions of 21)	Ab-C -1 E-G [#] 23 C-E -1	Db-F 23 A-C [#] -1 F-A -1	F [#] -A [#] 23 D-F [#] -1 B _b -D -1	B-D [#] 23 G-B -1 Eb-G -1
--	--	--	--	--

Johann Georg Neidhardt, 1732

Neidhardt (1732) Example #2 for chapter VII (before 3rd-circles)												
	E_b	B_b	F	C	G	D	A	E	B	F_#	C_#	G_#
Fifth in TU	-60	0	60	-120	-180	-180	-180	0	0	-60	0	0
Fifth in PC	- 1/12		1/12	- 1/6	- 1/4	- 1/4	- 1/4			- 1/12		
Fifth in SC	- 1/11		1/11	- 2/11	- 3/11	- 3/11	- 3/11			- 1/11		
Maj3rd % SC	81.8%	63.6%	36.4%	0.0%	18.2%	45.5%	63.6%	90.9%	90.9%	81.8%	90.9%	100.0%
Diesis portion	42.9%	33.3%	19.0%	0.0%	9.5%	23.8%	33.3%	47.6%	47.6%	42.9%	47.6%	52.4%
Min3rd % SC	-90.9%	-90.9%	-90.9%	-100.0%	-90.9%	-63.6%	-27.3%	-18.2%	-45.5%	-72.7%	-90.9%	-90.9%
Tone c	202.0	205.9	202.0	194.1	192.2	192.2	198.0	203.9	202.0	202.0	203.9	202.0
Semitone c	92.2	94.1	94.1	90.2	94.1	100.0	103.9	109.8	111.7	107.8	103.9	98.0
Enharmonic	<i>D_#</i>	<i>A_#</i>	<i>E_#</i>	<i>B_#</i>	<i>F_x</i>	<i>C_x</i>	<i>B_{bb}</i>	<i>F_b</i>	<i>C_b</i>	<i>G_b</i>	<i>D_b</i>	<i>A_b</i>
Enharm err % PC	50.0%	58.3%	75.0%	100.0%	100.0%	91.7%	-116.7%	-125.0%	-108.3%	-91.7%	-83.3%	-66.7%
d 1/6PC (C) c	-11.7	-9.8	-5.9	0.0	0.0	-2.0	-3.9	-5.9	-2.0	2.0	3.9	7.8
Error % PC	-50.0%	-41.7%	-25.0%	0.0%	0.0%	-8.3%	-16.7%	-25.0%	-8.3%	8.3%	16.7%	33.3%
	E_b	B_b	F	C	G	D	A	E	B	F_#	C_#	G_#
d 1/6SC (C) c	-10.8	-9.1	-5.5	0.0	-0.3	-2.6	-4.9	-7.2	-3.6	0.0	1.6	5.2
d ET (C) c	-5.9	-5.9	-3.9	0.0	-2.0	-5.9	-9.8	-13.7	-11.7	-9.8	-9.8	-7.8
d ET (A) c	3.9	3.9	5.9	9.8	7.8	3.9	0.0	-3.9	-2.0	0.0	0.0	2.0

Major tetrachord	E _b -F-G-A _b	B _b -C-D-E _b	F-G-A-B _b	C-D-E-F	G-A-B-C	D-E-F _# -G	A-B-C _# -D	E-F _# -G _# -A	B-C _# -D _# -E	F _# -G _# -A _# -B	D _b -E _b -F-G _b	A _b -B _b -C-D _b
Ut-Re	202.0	205.9	202.0	194.1	192.2	192.2	198.0	203.9	202.0	202.0	203.9	202.0
Re-Mi	202.0	194.1	192.2	192.2	198.0	203.9	202.0	202.0	203.9	202.0	202.0	205.9
Mi-Fa	94.1	100.0	103.9	109.8	111.7	107.8	103.9	98.0	92.2	94.1	94.1	90.2
Minor tetrachord	E _b -F-G _b -A _b	B _b -C-D _b -E _b	F-G-A _b -B _b	C-D-E _b -F	G-A-B _b -C	D-E-F-G	A-B-C-D	E-F _# -G-A	B-C _# -D-E	F _# -G _# -A-B	C _# -D _# -E-F _#	G _# -A _# -B-C _#
Re-Mi	202.0	205.9	202.0	194.1	192.2	192.2	198.0	203.9	202.0	202.0	203.9	202.0
Mi-Fa	94.1	90.2	94.1	100.0	103.9	109.8	111.7	107.8	103.9	98.0	92.2	94.1
Fa-Sol	202.0	203.9	202.0	202.0	205.9	202.0	194.1	192.2	192.2	198.0	203.9	202.0

Handling diesis	Ab-C 11	Db-F 10	F _# -A _# 9	B-D _# 10
within the octave	E-G _# 10	A-C _# 7	D-F _# 5	G-B 2
(portions of 21)	C-E 0	F-A 4	B _b -D 7	E _b -G 9

Johann Georg Neidhardt, 1732

Neidhardt (1732) Example #3 for chapter VII (before 3rd-circles)												
	E_b	B_b	F	C	G	D	A	E	B	F[#]	C[#]	G[#]
Fifth in TU	-60	0	0	-120	-120	-180	-180	0	0	-60	0	0
Fifth in PC	- 1/12			- 1/6	- 1/6	- 1/4	- 1/4			- 1/12		
Fifth in SC	- 1/11			- 2/11	- 2/11	- 3/11	- 3/11			- 1/11		
Maj3rd % SC	72.7%	63.6%	36.4%	9.1%	27.3%	45.5%	63.6%	90.9%	90.9%	81.8%	90.9%	90.9%
Diesis portion	38.1%	33.3%	19.0%	4.8%	14.3%	23.8%	33.3%	47.6%	47.6%	42.9%	47.6%	47.6%
Min3rd % SC	-90.9%	-90.9%	-90.9%	-90.9%	-81.8%	-63.6%	-36.4%	-27.3%	-45.5%	-72.7%	-90.9%	-90.9%
Tone c	202.0	203.9	200.0	196.1	194.1	192.2	198.0	203.9	202.0	202.0	203.9	202.0
Semitone c	92.2	94.1	94.1	92.2	96.1	100.0	103.9	109.8	109.8	105.9	103.9	98.0
Enharmonic	<i>D[#]</i>	<i>A[#]</i>	<i>E[#]</i>	<i>B[#]</i>	<i>F_x</i>	<i>C_x</i>	<i>B_{bb}</i>	<i>F_b</i>	<i>C_b</i>	<i>G_b</i>	<i>D_b</i>	<i>A_b</i>
Enharm err % PC	58.3%	66.7%	83.3%	100.0%	100.0%	100.0%	-108.3%	-116.7%	-100.0%	-83.3%	-75.0%	-58.3%
d 1/6PC (C) c	-9.8	-7.8	-3.9	0.0	0.0	0.0	-2.0	-3.9	0.0	3.9	5.9	9.8
Error % PC	-41.7%	-33.3%	-16.7%	0.0%	0.0%	0.0%	-8.3%	-16.7%	0.0%	16.7%	25.0%	41.7%
	E_b	B_b	F	C	G	D	A	E	B	F[#]	C[#]	G[#]
d 1/6SC (C) c	-8.8	-7.2	-3.6	0.0	-0.3	-0.7	-2.9	-5.2	-1.6	2.0	3.6	7.2
d ET (C) c	-3.9	-3.9	-2.0	0.0	-2.0	-3.9	-7.8	-11.7	-9.8	-7.8	-7.8	-5.9
d ET (A) c	3.9	3.9	5.9	7.8	5.9	3.9	0.0	-3.9	-2.0	0.0	0.0	2.0

Major tetrachord	E _b -F-G-A _b	B _b -C-D-E _b	F-G-A-B _b	C-D-E-F	G-A-B-C	D-E-F [#] -G	A-B-C [#] -D	E-F [#] -G [#] -A	B-C [#] -D [#] -E	F [#] -G [#] -A [#] -B	D _b -E _b -F-G _b	A _b -B _b -C-D _b
Ut-Re	202.0	203.9	200.0	196.1	194.1	192.2	198.0	203.9	202.0	202.0	203.9	202.0
Re-Mi	200.0	196.1	194.1	192.2	198.0	203.9	202.0	202.0	203.9	202.0	202.0	203.9
Mi-Fa	96.1	100.0	103.9	109.8	109.8	105.9	103.9	98.0	92.2	94.1	94.1	92.2
Minor tetrachord	E _b -F-G _b -A _b	B _b -C-D _b -E _b	F-G-A _b -B _b	C-D-E _b -F	G-A-B _b -C	D-E-F-G	A-B-C-D	E-F [#] -G-A	B-C [#] -D-E	F [#] -G [#] -A-B	C [#] -D [#] -E-F [#]	G [#] -A [#] -B-C [#]
Re-Mi	202.0	203.9	200.0	196.1	194.1	192.2	198.0	203.9	202.0	202.0	203.9	202.0
Mi-Fa	94.1	92.2	96.1	100.0	103.9	109.8	109.8	105.9	103.9	98.0	92.2	94.1
Fa-Sol	202.0	203.9	202.0	202.0	203.9	200.0	196.1	194.1	192.2	198.0	203.9	202.0

Handling diesis	Ab-C 10	Db-F 10	F [#] -A [#] 9	B-D [#] 10
within the octave	E-G [#] 10	A-C [#] 7	D-F [#] 5	G-B 3
(portions of 21)	C-E 1	F-A 4	B _b -D 7	E _b -G 8

Johann Georg Neidhardt, 1732

Neidhardt then presents a chart with five 3rd-circle temperament examples:

Neidhardt (1732) 3rd-circle #1 (Village)												
	Eb	Bb	F	C	G	D	A	E	B	F#	C#	G#
Fifth in TU	0	-60	0	-60	-120	-180	-180	0	-60	0	-60	0
Fifth in PC		- 1/12		- 1/12	- 1/6	- 1/4	- 1/4		- 1/12		- 1/12	
Fifth in SC		- 1/11		- 1/11	- 2/11	- 3/11	- 3/11		- 1/11		- 1/11	
Maj3rd % SC	81.8%	63.6%	45.5%	18.2%	27.3%	36.4%	63.6%	81.8%	81.8%	90.9%	81.8%	90.9%
Diesis portion	42.9%	33.3%	23.8%	9.5%	14.3%	19.0%	33.3%	42.9%	42.9%	47.6%	42.9%	47.6%
Min3rd % SC	-90.9%	-90.9%	-90.9%	-90.9%	-81.8%	-72.7%	-45.5%	-27.3%	-45.5%	-63.6%	-90.9%	-81.8%
Tone c	202.0	202.0	202.0	198.0	194.1	192.2	198.0	202.0	202.0	202.0	202.0	203.9
Semitone c	94.1	94.1	94.1	94.1	94.1	98.0	103.9	107.8	107.8	107.8	103.9	100.0
Enharmonic	<i>D#</i>	<i>A#</i>	<i>E#</i>	<i>B#</i>	<i>Fx</i>	<i>Cx</i>	<i>Bbb</i>	<i>Fb</i>	<i>Cb</i>	<i>Gb</i>	<i>Db</i>	<i>Ab</i>
Enharm err % PC	58.3%	75.0%	83.3%	100.0%	108.3%	108.3%	-100.0%	-108.3%	-91.7%	-83.3%	-66.7%	-58.3%
d 1/6PC (C) c	-9.8	-5.9	-3.9	0.0	2.0	2.0	0.0	-2.0	2.0	3.9	7.8	9.8
Error % PC	-41.7%	-25.0%	-16.7%	0.0%	8.3%	8.3%	0.0%	-8.3%	8.3%	16.7%	33.3%	41.7%
Primary function	Eb	Bb	F	C	G	D	A	E	B	F#	C#	G#
d 1/6SC (C) c	-8.8	-5.2	-3.6	0.0	1.6	1.3	-1.0	-3.3	0.3	2.0	5.5	7.2
d ET (C) c	-3.9	-2.0	-2.0	0.0	0.0	-2.0	-5.9	-9.8	-7.8	-7.8	-5.9	-5.9
d ET (A) c	2.0	3.9	3.9	5.9	5.9	3.9	0.0	-3.9	-2.0	-2.0	0.0	0.0

Major tetrachord	Eb-F-G-Ab	Bb-C-D-Eb	F-G-A-Bb	C-D-E-F	G-A-B-C	D-E-F#-G	A-B-C#-D	E-F#-G#-A	B-C#-D#-E	F#-G#-A#-B	Db-Eb-F-Gb	Ab-Bb-C-Db
Ut-Re	202.0	202.0	202.0	198.0	194.1	192.2	198.0	202.0	202.0	202.0	202.0	203.9
Re-Mi	202.0	198.0	194.1	192.2	198.0	202.0	202.0	202.0	202.0	203.9	202.0	202.0
Mi-Fa	94.1	98.0	103.9	107.8	107.8	107.8	103.9	100.0	94.1	94.1	94.1	94.1
Minor tetrachord	Eb-F-Gb-Ab	Bb-C-Db-Eb	F-G-Ab-Bb	C-D-Eb-F	G-A-Bb-C	D-E-F-G	A-B-C-D	E-F#-G-A	B-C#-D-E	F#-G#-A-B	C#-D#-E-F#	G#-A#-B-C#
Re-Mi	202.0	202.0	202.0	198.0	194.1	192.2	198.0	202.0	202.0	202.0	202.0	203.9
Mi-Fa	94.1	94.1	94.1	98.0	103.9	107.8	107.8	107.8	103.9	100.0	94.1	94.1
Fa-Sol	202.0	202.0	203.9	202.0	202.0	202.0	198.0	194.1	192.2	198.0	202.0	202.0

Handling diesis	Ab-C 10	Db-F 9	F#-A# 10	B-D# 9
within the octave	E-G# 9	A-C# 7	D-F# 4	G-B 3
(portions of 21)	C-E 2	F-A 5	Bb-D 7	Eb-G 9

Johann Georg Neidhardt, 1732

Neidhardt (1732) 3rd-circle #2 (Small City)

	E_b	B_b	F	C	G	D	A	E	B	F[#]	C[#]	G[#]
Fifth in TU	-60	0	0	-120	-120	-120	-120	-60	-60	0	0	-60
Fifth in PC	- 1/12			- 1/6	- 1/6	- 1/6	- 1/6	- 1/12	- 1/12			- 1/12
Fifth in SC	- 1/11			- 2/11	- 2/11	- 2/11	- 2/11	- 1/11	- 1/11			- 1/11
Maj3rd % SC	72.7%	63.6%	45.5%	27.3%	36.4%	45.5%	63.6%	81.8%	81.8%	81.8%	81.8%	81.8%
Diesis portion	38.1%	33.3%	23.8%	14.3%	19.0%	23.8%	33.3%	42.9%	42.9%	42.9%	42.9%	42.9%
Min3rd % SC	-90.9%	-81.8%	-81.8%	-90.9%	-81.8%	-63.6%	-45.5%	-45.5%	-54.5%	-63.6%	-81.8%	-90.9%
Tone c	202.0	203.9	200.0	196.1	196.1	196.1	198.0	200.0	202.0	203.9	202.0	200.0
Semitone c	96.1	96.1	94.1	94.1	98.0	100.0	102.0	105.9	107.8	105.9	102.0	98.0
Enharmonic	<i>D[#]</i>	<i>A[#]</i>	<i>E[#]</i>	<i>B[#]</i>	<i>F_x</i>	<i>C_x</i>	<i>B_{bb}</i>	<i>F_b</i>	<i>C_b</i>	<i>G_b</i>	<i>D_b</i>	<i>A_b</i>
Enharm err % PC	58.3%	66.7%	83.3%	100.0%	100.0%	100.0%	-100.0%	-100.0%	-91.7%	-83.3%	-66.7%	-50.0%
d 1/6PC (C) c	-9.8	-7.8	-3.9	0.0	0.0	0.0	0.0	0.0	2.0	3.9	7.8	11.7
Error % PC	-41.7%	-33.3%	-16.7%	0.0%	0.0%	0.0%	0.0%	0.0%	8.3%	16.7%	33.3%	50.0%
Primary function	E_b	B_b	F	C	G	D	A	E	B	F[#]	C[#]	G[#]
d 1/6SC (C) c	-8.8	-7.2	-3.6	0.0	-0.3	-0.7	-1.0	-1.3	0.3	2.0	5.5	9.1
d ET (C) c	-3.9	-3.9	-2.0	0.0	-2.0	-3.9	-5.9	-7.8	-7.8	-7.8	-5.9	-3.9
d ET (A) c	2.0	2.0	3.9	5.9	3.9	2.0	0.0	-2.0	-2.0	-2.0	0.0	2.0

Major tetrachord	E _b -F-G-A _b	B _b -C-D-E _b	F-G-A-B _b	C-D-E-F	G-A-B-C	D-E-F [#] -G	A-B-C [#] -D	E-F [#] -G [#] -A	B-C [#] -D [#] -E	F [#] -G [#] -A [#] -B	D _b -E _b -F-G _b	A _b -B _b -C-D _b
Ut-Re	202.0	203.9	200.0	196.1	196.1	196.1	198.0	200.0	202.0	203.9	202.0	200.0
Re-Mi	200.0	196.1	196.1	196.1	198.0	200.0	202.0	203.9	202.0	200.0	202.0	203.9
Mi-Fa	98.0	100.0	102.0	105.9	107.8	105.9	102.0	98.0	96.1	96.1	94.1	94.1
Minor tetrachord	E _b -F-G _b -A _b	B _b -C-D _b -E _b	F-G-A _b -B _b	C-D-E _b -F	G-A-B _b -C	D-E-F-G	A-B-C-D	E-F [#] -G-A	B-C [#] -D-E	F [#] -G [#] -A-B	C [#] -D [#] -E-F [#]	G [#] -A [#] -B-C [#]
Re-Mi	202.0	203.9	200.0	196.1	196.1	196.1	198.0	200.0	202.0	203.9	202.0	200.0
Mi-Fa	94.1	94.1	98.0	100.0	102.0	105.9	107.8	105.9	102.0	98.0	96.1	96.1
Fa-Sol	203.9	202.0	200.0	202.0	203.9	200.0	196.1	196.1	196.1	198.0	200.0	202.0

Handling diesis	A _b -C 9	D _b -F 9	F [#] -A [#] 9	B-D [#] 9
within the octave	E-G [#] 9	A-C [#] 7	D-F [#] 5	G-B 4
(portions of 21)	C-E 3	F-A 5	B _b -D 7	E _b -G 8



Johann Georg Neidhardt, 1732

Neidhardt (1732) 3rd-circle #3												
	E_b	B_b	F	C	G	D	A	E	B	F_#	C_#	G_#
Fifth in TU	120	-120	-60	-120	-120	-60	-120	-120	120	-120	-60	-60
Fifth in PC	1/6	- 1/6	- 1/12	- 1/6	- 1/6	- 1/12	- 1/6	- 1/6	1/6	- 1/6	- 1/12	- 1/12
Fifth in SC	2/11	- 2/11	- 1/11	- 2/11	- 2/11	- 1/11	- 2/11	- 2/11	2/11	- 2/11	- 1/11	- 1/11
Maj3rd % SC	72.7%	36.4%	45.5%	36.4%	36.4%	72.7%	63.6%	72.7%	81.8%	81.8%	81.8%	81.8%
Diesis portion	38.1%	19.0%	23.8%	19.0%	19.0%	38.1%	33.3%	38.1%	42.9%	42.9%	42.9%	42.9%
Min3rd % SC	-63.6%	-100.0%	-90.9%	-90.9%	-54.5%	-54.5%	-54.5%	-54.5%	-54.5%	-81.8%	-81.8%	-90.9%
Tone c	203.9	198.0	198.0	196.1	198.0	198.0	196.1	203.9	203.9	198.0	200.0	205.9
Semitone c	98.0	90.2	98.0	96.1	98.0	100.0	105.9	105.9	107.8	100.0	100.0	100.0
Enharmonic	<i>D_#</i>	<i>A_#</i>	<i>E_#</i>	<i>B_#</i>	<i>F_x</i>	<i>C_x</i>	<i>B_{bb}</i>	<i>F_b</i>	<i>C_b</i>	<i>G_b</i>	<i>D_b</i>	<i>A_b</i>
Enharm err % PC	58.3%	91.7%	91.7%	100.0%	100.0%	100.0%	-91.7%	-91.7%	-91.7%	-58.3%	-58.3%	-50.0%
d 1/6PC (C) c	-9.8	-2.0	-2.0	0.0	0.0	0.0	2.0	2.0	2.0	9.8	9.8	11.7
Error % PC	-41.7%	-8.3%	-8.3%	0.0%	0.0%	0.0%	8.3%	8.3%	8.3%	41.7%	41.7%	50.0%
Primary function	E_b	B_b	F	C	G	D	A	E	B	F_#	C_#	G_#
d 1/6SC (C) c	-8.8	-1.3	-1.6	0.0	-0.3	-0.7	1.0	0.7	0.3	7.8	7.5	9.1
d ET (C) c	-3.9	2.0	0.0	0.0	-2.0	-3.9	-3.9	-5.9	-7.8	-2.0	-3.9	-3.9
d ET (A) c	0.0	5.9	3.9	3.9	2.0	0.0	0.0	-2.0	-3.9	2.0	0.0	0.0

Major tetrachord	E _b -F-G-A _b	B _b -C-D-E _b	F-G-A-B _b	C-D-E-F	G-A-B-C	D-E-F _# -G	A-B-C _# -D	E-F _# -G _# -A	B-C _# -D _# -E	F _# -G _# -A _# -B	D _b -E _b -F-G _b	A _b -B _b -C-D _b
Ut-Re	203.9	198.0	198.0	196.1	198.0	198.0	196.1	203.9	203.9	198.0	200.0	205.9
Re-Mi	198.0	196.1	198.0	198.0	196.1	203.9	203.9	198.0	200.0	205.9	203.9	198.0
Mi-Fa	98.0	100.0	105.9	105.9	107.8	100.0	100.0	100.0	98.0	90.2	98.0	96.1
Minor tetrachord	E _b -F-G _b -A _b	B _b -C-D _b -E _b	F-G-A _b -B _b	C-D-E _b -F	G-A-B _b -C	D-E-F-G	A-B-C-D	E-F _# -G-A	B-C _# -D-E	F _# -G _# -A-B	C _# -D _# -E-F _#	G _# -A _# -B-C _#
Re-Mi	203.9	198.0	198.0	196.1	198.0	198.0	196.1	203.9	203.9	198.0	200.0	205.9
Mi-Fa	98.0	96.1	98.0	100.0	105.9	105.9	107.8	100.0	100.0	100.0	98.0	90.2
Fa-Sol	198.0	200.0	205.9	203.9	198.0	198.0	196.1	198.0	198.0	196.1	203.9	203.9

Handling diesis	A _b -C 9	D _b -F 9	F _# -A _# 9	B-D _# 9
within the octave	E-G _# 8	A-C _# 7	D-F _# 8	G-B 4
(portions of 21)	C-E 4	F-A 5	B _b -D 4	E _b -G 8

Johann Georg Neidhardt, 1732

Neidhardt (1732) 3rd-circle #4

	E_b	B_b	F	C	G	D	A	E	B	F[#]	C[#]	G[#]
Fifth in TU	60	-120	0	-120	-120	-120	0	-120	0	-60	-60	-60
Fifth in PC	1/12	- 1/6		- 1/6	- 1/6	- 1/6		- 1/6		- 1/12	- 1/12	- 1/12
Fifth in SC	1/11	- 2/11		- 2/11	- 2/11	- 2/11		- 2/11		- 1/11	- 1/11	- 1/11
Maj3rd % SC	72.7%	45.5%	45.5%	45.5%	45.5%	63.6%	72.7%	63.6%	72.7%	81.8%	72.7%	81.8%
Diesis portion	38.1%	23.8%	23.8%	23.8%	23.8%	33.3%	38.1%	33.3%	38.1%	42.9%	38.1%	42.9%
Min3rd % SC	-72.7%	-90.9%	-81.8%	-90.9%	-63.6%	-63.6%	-45.5%	-63.6%	-63.6%	-81.8%	-72.7%	-81.8%
Tone c	202.0	200.0	200.0	196.1	196.1	200.0	200.0	200.0	202.0	200.0	200.0	203.9
Semitone c	100.0	94.1	98.0	96.1	98.0	100.0	105.9	102.0	105.9	102.0	100.0	98.0
Enharmonic	<i>D[#]</i>	<i>A[#]</i>	<i>E[#]</i>	<i>B[#]</i>	<i>F_x</i>	<i>C_x</i>	<i>B_{bb}</i>	<i>F_b</i>	<i>C_b</i>	<i>G_b</i>	<i>D_b</i>	<i>A_b</i>
Enharm err % PC	58.3%	83.3%	83.3%	100.0%	100.0%	100.0%	-100.0%	-83.3%	-83.3%	-66.7%	-58.3%	-50.0%
d 1/6PC (C) c	-9.8	-3.9	-3.9	0.0	0.0	0.0	0.0	3.9	3.9	7.8	9.8	11.7
Error % PC	-41.7%	-16.7%	-16.7%	0.0%	0.0%	0.0%	0.0%	16.7%	16.7%	33.3%	41.7%	50.0%
Primary function	E_b	B_b	F	C	G	D	A	E	B	F[#]	C[#]	G[#]
d 1/6SC (C) c	-8.8	-3.3	-3.6	0.0	-0.3	-0.7	-1.0	2.6	2.3	5.9	7.5	9.1
d ET (C) c	-3.9	0.0	-2.0	0.0	-2.0	-3.9	-5.9	-3.9	-5.9	-3.9	-3.9	-3.9
d ET (A) c	2.0	5.9	3.9	5.9	3.9	2.0	0.0	2.0	0.0	2.0	2.0	2.0

Major tetrachord	E _b -F-G-A _b	B _b -C-D-E _b	F-G-A-B _b	C-D-E-F	G-A-B-C	D-E-F [#] -G	A-B-C [#] -D	E-F [#] -G [#] -A	B-C [#] -D [#] -E	F [#] -G [#] -A [#] -B	D _b -E _b -F-G _b	A _b -B _b -C-D _b
Ut-Re	202.0	200.0	200.0	196.1	196.1	200.0	200.0	200.0	202.0	200.0	200.0	203.9
Re-Mi	200.0	196.1	196.1	200.0	200.0	200.0	202.0	200.0	200.0	203.9	202.0	200.0
Mi-Fa	98.0	100.0	105.9	102.0	105.9	102.0	100.0	98.0	100.0	94.1	98.0	96.1
Minor tetrachord	E _b -F-G _b -A _b	B _b -C-D _b -E _b	F-G-A _b -B _b	C-D-E _b -F	G-A-B _b -C	D-E-F-G	A-B-C-D	E-F [#] -G-A	B-C [#] -D-E	F [#] -G [#] -A-B	C [#] -D [#] -E-F [#]	G [#] -A [#] -B-C [#]
Re-Mi	202.0	200.0	200.0	196.1	196.1	200.0	200.0	200.0	202.0	200.0	200.0	203.9
Mi-Fa	98.0	96.1	98.0	100.0	105.9	102.0	105.9	102.0	100.0	98.0	100.0	94.1
Fa-Sol	200.0	200.0	203.9	202.0	200.0	200.0	196.1	196.1	200.0	200.0	200.0	202.0

Handling diesis within the octave (portions of 21)	A _b -C 9 E-G [#] 7 C-E 5	D _b -F 8 A-C [#] 8 F-A 5	F [#] -A [#] 9 D-F [#] 7 B _b -D 5	B-D [#] 8 G-B 5 E _b -G 8
--	---	---	--	---



Johann Georg Neidhardt, 1732

Neidhardt (1732) 3rd-circle #5

	E_b	B_b	F	C	G	D	A	E	B	F[#]	C[#]	G[#]
Fifth in TU	-60	0	-120	-60	-60	-60	-120	-60	-60	0	-60	-60
Fifth in PC	- 1/12		- 1/6	- 1/12	- 1/12	- 1/12	- 1/6	- 1/12	- 1/12		- 1/12	- 1/12
Fifth in SC	- 1/11		- 2/11	- 1/11	- 1/11	- 1/11	- 2/11	- 1/11	- 1/11		- 1/11	- 1/11
Maj3rd % SC	63.6%	63.6%	54.5%	54.5%	54.5%	54.5%	63.6%	72.7%	72.7%	72.7%	72.7%	63.6%
Diesis portion	33.3%	33.3%	28.6%	28.6%	28.6%	28.6%	33.3%	38.1%	38.1%	38.1%	38.1%	33.3%
Min3rd % SC	-81.8%	-72.7%	-81.8%	-72.7%	-72.7%	-63.6%	-72.7%	-63.6%	-63.6%	-63.6%	-81.8%	-81.8%
Tone c	202.0	200.0	198.0	200.0	200.0	198.0	198.0	200.0	202.0	202.0	200.0	200.0
Semitone c	98.0	98.0	96.1	100.0	100.0	100.0	100.0	103.9	102.0	102.0	100.0	100.0
Enharmonic	<i>D[#]</i>	<i>A[#]</i>	<i>E[#]</i>	<i>B[#]</i>	<i>F_x</i>	<i>C_x</i>	<i>B_{bb}</i>	<i>F_b</i>	<i>C_b</i>	<i>G_b</i>	<i>D_b</i>	<i>A_b</i>
Enharm err % PC	75.0%	83.3%	100.0%	100.0%	108.3%	116.7%	-75.0%	-75.0%	-66.7%	-58.3%	-41.7%	-33.3%
d 1/6PC (C) c	-5.9	-3.9	0.0	0.0	2.0	3.9	5.9	5.9	7.8	9.8	13.7	15.6
Error % PC	-25.0%	-16.7%	0.0%	0.0%	8.3%	16.7%	25.0%	25.0%	33.3%	41.7%	58.3%	66.7%
Primary function	E_b	B_b	F	C	G	D	A	E	B	F[#]	C[#]	G[#]
d 1/6SC (C) c	-4.9	-3.3	0.3	0.0	1.6	3.3	4.9	4.6	6.2	7.8	11.4	13.0
d ET (C) c	0.0	0.0	2.0	0.0	0.0	0.0	0.0	-2.0	-2.0	-2.0	0.0	0.0
d ET (A) c	0.0	0.0	2.0	0.0	0.0	0.0	0.0	-2.0	-2.0	-2.0	0.0	0.0

Major tetrachord	E _b -F-G-A _b	B _b -C-D-E _b	F-G-A-B _b	C-D-E-F	G-A-B-C	D-E-F [#] -G	A-B-C [#] -D	E-F [#] -G [#] -A	B-C [#] -D [#] -E	F [#] -G [#] -A [#] -B	D _b -E _b -F-G _b	A _b -B _b -C-D _b
Ut-Re	202.0	200.0	198.0	200.0	200.0	198.0	198.0	200.0	202.0	202.0	200.0	200.0
Re-Mi	198.0	200.0	200.0	198.0	198.0	200.0	202.0	202.0	200.0	200.0	202.0	200.0
Mi-Fa	100.0	100.0	100.0	103.9	102.0	102.0	100.0	100.0	98.0	98.0	96.1	100.0
Minor tetrachord	E _b -F-G _b -A _b	B _b -C-D _b -E _b	F-G-A _b -B _b	C-D-E _b -F	G-A-B _b -C	D-E-F-G	A-B-C-D	E-F [#] -G-A	B-C [#] -D-E	F [#] -G [#] -A-B	C [#] -D [#] -E-F [#]	G [#] -A [#] -B-C [#]
Re-Mi	202.0	200.0	198.0	200.0	200.0	198.0	198.0	200.0	202.0	202.0	200.0	200.0
Mi-Fa	96.1	100.0	100.0	100.0	100.0	103.9	102.0	102.0	100.0	100.0	98.0	98.0
Fa-Sol	202.0	200.0	200.0	202.0	200.0	198.0	200.0	200.0	198.0	198.0	200.0	202.0

Handling diesis within the octave (portions of 21)	A _b -C 7	D _b -F 8	F [#] -A [#] 8	B-D [#] 8
	E-G [#] 8	A-C [#] 7	D-F [#] 6	G-B 6
	C-E 6	F-A 6	B _b -D 7	E _b -G 7

